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DESIGN OF A HEATER FOR DRYING ARTICLES MADE OF CURRENT-CONDUCTING MATERIALS

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We present a description and calculation of the temperature and moisture-content fields of capillary-porous materials in the process of heating them by an electric current with removal of hygroscopic moisture.

Use a volumetric method of heat supply considerably increases the efficiency of heat treatment of articles, including drying. The electrical-conduction properties of some materials allow heating them without applying secondary heat sources. The temperature difference over the specimen cross section appears in the course of electric heating, favors transfer of moisture from its central layers to the periphery [1, p. 319]. When the heating is sufficiently intense, a filtrational flux appears that is attributable to the total-pressure gradient and is likewise directed to the surface. All this increases the economic performance figures of drying and is indicative of the prospects for using the electrical-contact method of heating [2].

The distribution of moisture in an article being treated is determined by the temperature field, for whose description the Fourier differential equation of heat conduction is used [3].

If we neglect the influence of heat losses in contacts on the temperature distribution, we can write the heat-balance equation as

$$mC \frac{dt}{d\tau} + \alpha F (t - t_0) = AI^2 R . \quad (1)$$

Since the motion of the bound substance in a capillary-porous body is considered to be rather slow, the temperature of the liquid is virtually equal to that of the walls of the capillary [4]. Many parameters change in the course of heating: current, the resistance of the article, moisture content, thermal-conductivity coefficient, the heat capacity of the moist material, etc. It is virtually impossible to relate all these parameters to the temperature of the article, and therefore we divide the heating curve into a number of intervals within which the values of I, R, U, λ, C will be considered constant. Separating variables and integrating Eq. (1), we determine the time of heating the article for any calculational interval:

$$\tau_i = \frac{mC_i}{\alpha F} \ln \frac{AI_i^2 R - \alpha F (t_{\text{beg}} - t_0)}{AI_i^2 R - \alpha F (t_{\text{end}} - t_0)} . \quad (2)$$

The total time of heating is $\tau = \sum_{i=0}^{n} \tau_i$.

With consideration of the aforesaid, the differential equation of heat conduction in the case of the one-dimensional problem takes the form [3]

$$\frac{\partial t}{\partial \tau} = a \left(\frac{\partial^2 t}{\partial x^2} + \frac{\Gamma}{x} \frac{\partial t}{\partial x} \right) + \frac{q}{C\rho_0} . \quad (3)$$

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We will consider the process of heating a cylindrical article whose length greatly exceeds its radius; in this case the geometric parameter Γ is equal to unity.

The initial conditions are: the temperature over the entire cross section is the same and equal to

$$t|_{\tau=0} = t_{\text{beg}} . \tag{4}$$

The boundary conditions are: 1) boundary conditions of the third kind on the body surface:

$$-\lambda \frac{dt}{dr}\bigg|_{r=r_0} = \alpha (t - t_0) + \varepsilon^* r^* i^*_{\text{sur}} + C_{\text{vap}} i^*_{\text{sur}} t_{\text{sur}} , \tag{5}$$

where $\alpha = \alpha_{\text{rad}} + \alpha_{\text{conv}}$; α_{rad} can be calculated as [1]

$$\alpha_{\text{rad}} = \frac{c_{\text{red}} \left[\left(\frac{T}{100}\right)^4 - \left(\frac{T_0}{100}\right)^4 \right]}{t - t_0} ; \tag{6}$$

values of α_{conv} are given in [2, p. 23]; 2) the solution is bounded along the article axis ($r = 0$).

In passing a current through a cylindrical billet, the volumetric density of internal heat sources is distributed according to the law

$$q = \frac{AI^2 \omega \mu_{\text{mat}} \mu_0}{4\pi^2 r_0^2 \, |J_1 (\xi_{\text{sur}} r_0)|^2} \, |J_0 (\xi_1 r)|^2 , \tag{7}$$

where $\xi_1 = \sqrt{-j\omega\mu_{\text{mat}}\mu_0\sigma}$.

Let $a\tau/r_0^2 = \text{Fo}$, $r/r_0 = z$, $t - t_0 = \theta$; then Eq. (3) can be rewritten in a more convenient form:

$$\frac{\partial \theta}{\partial \text{Fo}} = \frac{\partial^2 \theta}{\partial z^2} + \frac{1}{z} \frac{\partial \theta}{\partial z} + k_1 \, |J_0 (\xi_1 z)|^2 , \tag{8}$$

where

$$k_1 = \frac{AI^2 \omega \mu_{\text{mat}} \mu_0}{4\pi^2 \lambda \, |J_1 (\xi_1)|^2} ; \quad \xi_1 = r_0 \sqrt{- j\omega \mu_{\text{mat}} \mu_0 \sigma} ,$$

with the initial conditions

$$\theta|_{\text{Fo}=0} = \theta_{\text{beg}} \tag{9}$$

and the boundary conditions:

$$1) \quad -\lambda \frac{d\theta}{dz}\bigg|_{z=1} = \alpha r_0 \theta + \varepsilon^* r^* r_0 \, i^*_{\text{sur}} + C_{\text{vap}} \, r_0 \, i^*_{\text{sur}} \, \theta , \tag{10}$$

$$2) \quad z = 0 \; - \; \text{the solution is bounded} . \tag{11}$$

The solution of Eq. (8) with initial and boundary conditions (9)-(11) is obtained in the form

$$\theta \frac{4k_1}{q_1^2} \left(\frac{1}{\text{Bi}} \nu_1 + \chi_1 + \chi_z \right) + \sum_{n=1}^{\infty} C_{\text{vap}} J_0 (\mu_n z) \exp (- \mu_n^2 \, \text{Fo}) =$$

$$= \theta \frac{4k_1}{q_1^2}\left(\frac{1}{\text{Bi}}\nu_1 + \chi_1 + \chi_z\right) + \sum_{n=1}^{\infty} \frac{2}{H_{\mu_n}}\left[\frac{J_1(\mu_n)\,(\theta_{\text{vap}} - C_3)}{\mu_n} + \right.$$

$$\left. + \frac{4k_1}{q_1^2}\psi(q_1; \mu_n)\right] J_0(\mu_n z)\exp(-\mu_n^2 \text{Fo}) . \tag{12}$$

The first term is the temperature distribution over the billet cross section in a steady-state regime. Here:

$$q_1 = r_0\sqrt{\omega\mu_{\text{mat}}\mu_0\sigma}\ ; \tag{13}$$

$$\text{Bi} = \frac{\alpha r_0}{\lambda}, \tag{14}$$

$$C_3 = \frac{4k_1}{q_1^2}\left(\frac{1}{\text{Bi}}\nu_1 + \chi_1\right); \tag{15}$$

$$H_{\mu_n} = J_0^2(\mu_n) + J_1^2(\mu_n)\ ; \tag{16}$$

$$\nu_1 = \sum_{p=0}^{\infty}\frac{\left(\frac{q_1}{2}\right)^{4p+2}}{(p\,!)^2\,(2p)\,!\,(4p+2)}\ ; \tag{17}$$

$$\chi_1 = \sum_{p=0}^{\infty}\frac{\left(\frac{q_1}{2}\right)^{4p+2}}{(p\,!)^2\,(2p)\,!\,(4p+2)^2}\ ; \tag{18}$$

$$\chi_z = \sum_{p=0}^{\infty}\frac{\left(\frac{q_1 z}{2}\right)^{4p+2}}{(p\,!)^2\,(2p)\,!\,(4p+2)^2}\ ; \tag{19}$$

$$\psi(q_1; \mu_n) = \sum_{p=0}^{\infty}\frac{\left(\frac{q_1}{2}\right)^{4p+2}}{(p\,!)^2\,(2p)\,!\,(4p+2)^2}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{2}\mu_n\right)^{2k}}{(k\,!)^2\,(4p+4+2k)} .$$

Values of the functions ν_1, χ, $\psi(q_1; \mu_n)$, φ_{11}, φ_{red}, $J_0(\mu_1)$, and $J_1(\mu_1)$ are given in the form of graphs in [2, 5].

In the calculation of the second temperature interval, the cross-sectional distribution of temperature obtained at the end of the first temperature interval is taken as the initial condition. The temperature in the i-th interval is calculated from the formula

$$\theta_i \frac{4k_{1i}}{q_{1i-1}^2}\left(\frac{1}{\text{Bi}}\nu_{1i} + \chi_{1i} + \chi_{zi}\right) + \sum_{n=1}^{\infty}\frac{2}{H_{\mu_{ni}}}\left[\frac{J_1(\mu_{ni})\,(\theta_{3i-1} - C_{3i})}{\mu_{ni}} + \right.$$

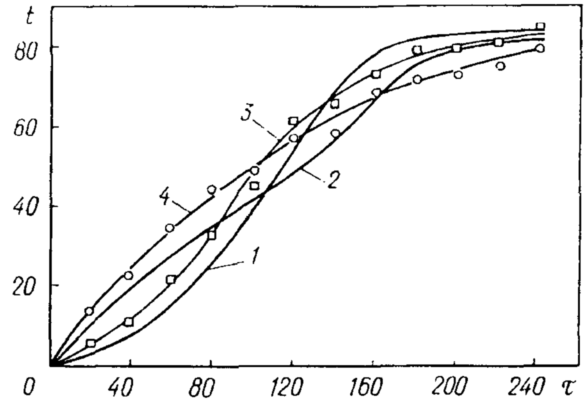

Fig. 1. Dependence of the temperature t of the surface and center of a graphite rod on the time of heating τ by an electrical-contact method. Curves 1 at the center and 2 on the surface correspond to calculated values; curves 3 and 4 correspond to experimental data on the temperature at the center and on the surface of the article, respectively: t, °C; r, sec.

$$+ \frac{4k_{1i-1}}{q_{1i-1}^2} \psi (q_{1i-1}; \mu_{ni}) + \frac{4k_{1i}}{q_{1i}^2} \psi (q_{1i}; \mu_{ni}) +$$

$$\left. + \sum_{p=1}^{\infty} C_{\text{vap}_{pi-1}} \exp (- \mu_{pi-1}^2 \text{Fo}_{i-1}) \varphi_{np} \right] J_0 (\mu_{ni} z) \exp (- \mu_{ni}^2 \text{Fo}) ,$$

where μ_n and μ_p are positive roots of the equation $\mu J(\mu) - \text{Bi} J_0(\mu) = 0$ [2, p. 25]. The series $\sum_{n=1}^{\infty} C_{\text{vap}} J_0(\mu_n z) \exp(-\mu_n^2 \text{Fo})$ converges very rapidly, and in practice it is sufficient to restrict ourselves to the first term of the series.

Results of calculations and an experimental check are given in Fig. 1. The specimens used were 10-mm-diameter graphite rods. Heating was used to remove hygroscopic moisture. It is evident that the discrepancy between the experimental and calculated data does not exceed 10–15%. To increase the intensity and safety of the process, the heating was done with simultaneous evacuation. Under these conditions the moisture in the material is transferred in the form of vapor [1, p. 147]. The density of the vapor flux into the chamber from the material surface i_{sur}^* is determined by the capacity of the vapor and gas removing system, i.e., a condenser and a vacuum pump. The moisture flux in the material, characterized by diffusion and effusion transport in the capillaries, is determined at any point of the cross section of the article [6, p. 89] by the formula

$$i^* = - \frac{DE\mu^*}{R^* T} \frac{\partial P}{\partial r} - 1.064 \sqrt{\left(\frac{\mu^*}{R^*}\right)} \frac{\partial}{\partial r} \left(\frac{P}{\sqrt{T}}\right) . \tag{20}$$

Substituting the value of the flux from the article surface i_{sur}^* into the left-hand side of Eq. (20), we obtain the boundary condition for the equation of mass conduction [1], which in our case has the form

$$\frac{\partial U}{\partial \tau} = a_m \left(\frac{\partial^2 U}{\partial x^2} + \frac{\Gamma}{x} \frac{\partial U}{\partial x}\right) + a_m \delta \left(\frac{\partial^2 t}{\partial x^2} + \frac{\Gamma}{x} \frac{\partial t}{\partial x}\right) . \tag{21}$$

The authors of [7] suggest an approximate solution of Eq. (21) that is based on the theory of deepening of the evaporation zone.

The specific consumption of electric energy for heating the article is calculated as the sum of specific consumptions in each interval:

$$W = \sum_{i=0}^{n} W_i , \qquad (22)$$

where $W_i = \tau_i(I_i^2 R_i + N_{10})$.

The proposed method can be used in designing installations for heat treatment and drying of piece and disperse electrically conducting materials and installations for regeneration of carbon adsorbents.

NOTATION

m, C, mass and heat capacity of the article; t, temperature at any point of the body; τ, time of heating; α, interval-mean coefficient of heat transfer from the article surface; F, surface area of heat transfer; t_0, temperature of the surrounding medium; A, electric equivalent of heat; I, current passed through the article; R, resistance of the moist material; i^*, density of the moisture flux; Γ, geometric parameter of the body; ρ_0, material density; q, density of internal heat sources; x, current coordinate; r, current radius of the article; r_0, total radius of the article; ε^*, vaporization criterion; r^*, heat of phase transition; C_{vap}, heat capacity of the vapor; c_{red}, reduced coefficient of emission; ω, angular frequency of the current; μ_{mat}, μ_0, magnetic permeability of the material and vacuum, respectively; $J_0(\mu)$ and $J_1(\mu)$, Bessel functions of the first kind and the zeroth and first orders, respectively; $j = \sqrt{-1}$, imaginary unit in a complex number; σ, specific electrical conductivity of the material; a, thermal diffusivity coefficient; Fo, Fourier number; Bi, Biot number; θ, temperature difference; z, current radius in relative units; λ, thermal-conductivity coefficient; P, vapor pressure; U, moisture content; R^*, universal gas constant; W, specific consumption of electric energy; E, resistance coefficient of vapor diffusion in the porous body; N_{10}, power of the transformer open circuit; μ^*, mass of the vapor in one mole; D, coefficient of vapor diffusion in air; a_m, coefficient of mass conduction. Subscripts: beg, beginning of the interval; end, end of the interval; rad, radiation; conv, convection; i, ordinal number of the temperature interval; vap, vapor; sur, surface; mat, material.

REFERENCES

1. A. V. Luikov, Theory of Drying [in Russian], Moscow (1968).
2. A. Kh. Valeev, in: Electrocontact Heating of Steel Billets (ed. by V. A. Grinfel'd) [in Russian], Moscow (1968).
3. A. V. Luikov, Theory of Heat Conduction [in Russian], Moscow (1952).
4. A. V. Luikov, Heat and Mass Transfer in the Processes of Drying [in Russian], Moscow (1956).
5. E. Janke and F. Emde, Tables of Functions [Russian translation], Moscow (1959).
6. V. V. Krasnikov, Conductive Drying [in Russian], Moscow (1973).
7. R. G. Safin, V. A. Lashkov, and L. G. Golubev, Inzh.-Fiz. Zh., 56, No. 2, 276-281 (1989).